Short Papers

Microwave Measurement Techniques for Below-Resonance Junction Circulators

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Abstract—**To have a full understanding of the experimental behavior of a junction circulator it is necessary to be able to measure the gyrator conductance, the susceptance slope parameter, and the frequencies of the two split resonant modes of the junction. It is also useful to be able to construct mode charts to determine the geometry of the device. The purpose of this short paper is to give simple methods by which each of these parameters may be measured in the case of the below-resonance stripline circulator. The techniques developed here all stem from the scattering and immittance matrices of the device. They are therefore quite general. They also require no phase information which makes them ideally suitable for reflectometer-type measurement. All measurements described here are made in the input transmission line of the device with the other two ports connected to similar transmission lines terminated in their characteristic impedance. The results obtained here apply to lossless circulators for which the two resonant modes are symmetrically split by the magnetic field, and for which the frequency variation of the third mode can be omitted.**

I. Introduction

Present measuring techniques for junction circulators include the direct measurement of scattering coefficients S_{11}, S_{12}, S_{13} [1] and eigenvalues s_1, s_{+1}, s_{-1} [2], [3] or the input gyrator admittance by first decoupling one port [4]–[7]. This latter measurement allows the equivalent circuit of the circulator to be verified.

In the network approach to junction circulators it may be more useful to obtain results in terms of the gyrator conductance, the susceptance slope parameter, and the difference between the two split frequencies of the magnetized junction. It is also desirable to be able to construct a mode chart of junction circulators. It is the purpose of this short paper to give simple methods by which each of these quantities may be measured one at a time.

The measurement techniques developed here are obtained by relating the scattering and impedance matrices of symmetrical junctions through their eigenvalues. They are therefore quite general. They also require no phase information which makes them suitable for reflectometer-type measurements. Furthermore, all measurements described in this short paper are made in the input transmission line of the junction with the other two ports connected to similar transmission lines terminated in their characteristic impedance. The results obtained here apply to lossless circulators for which the two resonant modes are symmetrically split by the magnetic field, and for which frequency variation of the third mode can be omitted. Such circulators include the conventional below-resonance stripline and the E-plane waveguide ones.

The short paper starts by relating the gyrator conductance at the center frequency of the circulator to the VSWR in the input transmission line of the junction with the other two transmission lines terminated in their characteristic impedance. This is done by relating the impedance and scattering matrices. The mode chart of circulators is next obtained by observing that the operating frequency of the device coincides with the one at which the VSWR passes through 2:1. It is next shown, with reference to the eigenvalue diagram of the scattering matrix, that for a lossless junction the two split frequencies always coincide with the two frequencies at which the VSWR passes through 2:1. The modes need not be distinguishable to obtain this result. Finally, the susceptance slope parameter, the gyrator conductance, and the splitting between the resonant modes are all related to the universal admittance equation. This allows any one of these three to be obtained by measuring the other two.

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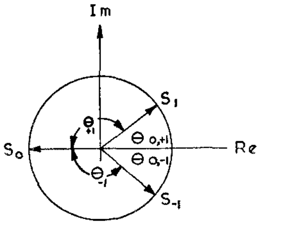

Fig. 1. Angle $\theta_{\pm 1}$ between s_0 and s_{+1}, s_{-1}.

Some references to the construction of mode charts have already appeared in the literature [8]–[11]. The split frequencies of a stripline circulator have been obtained by lightly coupling the junction to 50-Ω transmission lines by means of high-impedance quarter-wave transformers [12]. This ensures that the various modes of the disk structure are sharp and easily distinguished. They have also been obtained on a 3-port E-plane circulator from data on a 2-port junction [13].

II. Universal Definition of Input Admittance of Junction Circulators

Starting with the general form for the loaded Q-factor Q_L of a circulator one has for circulators which rely on 30° splitting between the degenerate modes (12).

$$\frac{1}{Q_L} = \sqrt{3}\left(\frac{\omega_{+1} - \omega_{-1}}{\omega_0}\right). \tag{1}$$

The above definition of the loaded Q-factor Q_L of the junction is consistent with the gyrator equation provided the general form for the normalized shunt conductance is [14]

$$g = \sqrt{3}\, b_1' \left(\frac{\omega_{+1} - \omega_{-1}}{\omega_0}\right). \tag{2}$$

Equation (2) may be taken as a universal definition of the gyrator conductance of junction circulators which rely on 30° splitting of the degenerate modes.

The total complex gyrator admittance is consistent with the definition of the loaded Q-factor provided it has the following general form:

$$y_{\text{in}} \approx \sqrt{3}\, b_1' \frac{(\omega_{+1} - \omega_{-1})}{\omega_0} + j2b_1' \frac{(\omega - \omega_0)}{\omega_0}. \tag{3}$$

The above equations apply to lossless circulators for which the splitting between the frequencies of the two degenerate resonant eigennetworks is symmetrical and for which the third eigennetwork is nonresonant. Such circulators include the below-resonance stripline and E-plane waveguide ones.

Here Q_L is the loaded Q-factor of the junction, ω_0 is the center frequency of the circulator, ω_{+1} and ω_{-1} are the split frequencies of the two resonant eigennetworks, g is the gyrator conductance, and b_1' is the susceptance slope parameter of the isotropic $n=1$ eigennetwork. One important property of this circuit is that the susceptance slope parameter is essentially determined by the geometry of the junction and remains nearly constant over fairly large variations in the gyrator conductance. It may therefore be derived from either the magnetized or demagnetized junction.

III. Measurement of Gyrator Conductance

A simple way in which the gyrator conductance of a junction circulator may be measured at the center frequency of the device will now be described. It consists of measuring the return loss or VSWR in the transmission line of one port with the other two ports connected to transmission lines terminated in their characteristic impedance. The approach used relies on the relation between the scattering coefficient S_{11} and the impedance one Z_{12}. At the center frequency of the device the normalized impedance coefficient Z_{12}

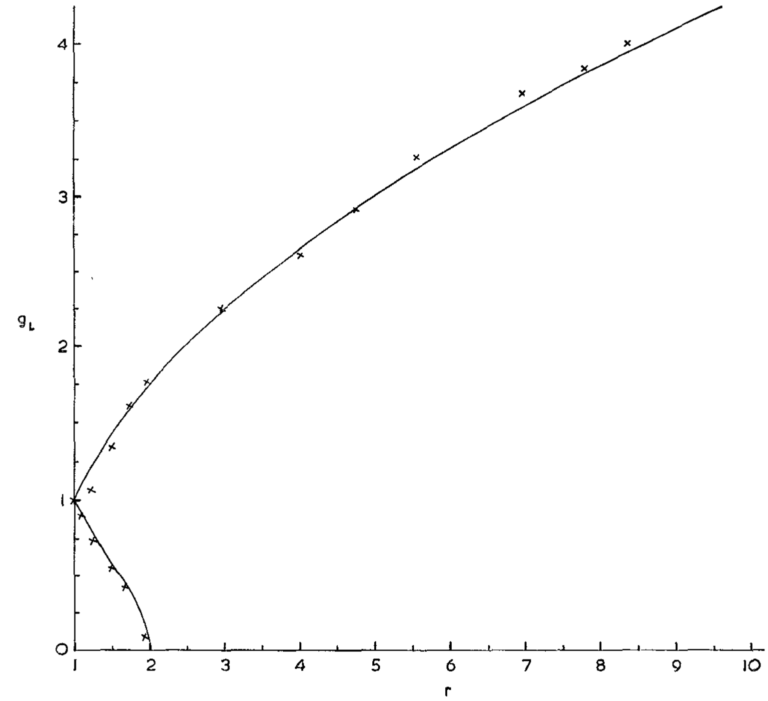

Fig. 2. Relation between r and g for magnetized junction.

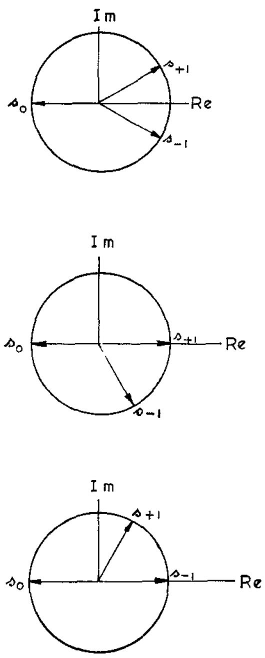

Fig. 3. Eigenvalue diagrams of an arbitrarily magnetized junction at ω_0, ω_{+1}, and ω_{-1}.

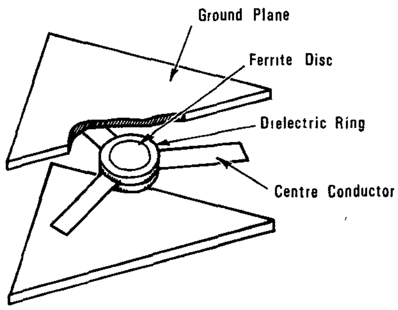

Fig. 4. Schematic of partially loaded ring stripline circulator.

is the gyrator one of the junction. The scattering coefficient S_{11} is given in terms of its eigenvalues by

$$S_{11} = \frac{s_0 + s_{+1} + s_{-1}}{3} \tag{4}$$

where for $S_{12} = -1$

$$s_0 = -1 \tag{5}$$

$$s_{+1} = s_0 e^{-j\theta_{+1}} \tag{6}$$

$$s_{-1} = s_0 e^{j\theta_{-1}}. \tag{7}$$

The angles $\theta_{\pm 1}$ are defined in Fig. 1. They normally lie between 0 and 180°. It is observed from this diagram that, in the complex plane, the resultant always lies along the real axis provided the splitting is symmetric $(\theta_{+1} = \theta_{-1})$. This means that the reference plane remains unchanged as $\theta_{\pm 1}$ is varied. In particular, it implies that the reference plane always coincides with that of an ideal circulator. This observation is in agreement with experiment in the case of the stripline junction.

Using the above relations S_{11} becomes

$$S_{11} = \frac{1 - 3\cot^2 \dfrac{\theta_{+1}}{2}}{3 + 3\cot^2 \dfrac{\theta_{+1}}{2}}. \tag{8}$$

For the normalized gyrator resistance z at the center frequency one has

$$z = \frac{Z_{12}}{Z_0} = \frac{z_0 + z_{+1}e^{(j2\pi)/3} + z_{-1}e^{(-j2\pi)/3}}{3} \tag{9}$$

where

$$z_0 = \frac{1 + s_0}{1 - s_0} = 0 \tag{10}$$

$$z_{+1} = \frac{1 + s_{+1}}{1 - s_{+1}} = j \tan \frac{\theta_{+1}}{2} \tag{11}$$

$$z_{-1} = \frac{1 + s_{-1}}{1 - s_{-1}} = -j \tan \frac{\theta_{+1}}{2}. \tag{12}$$

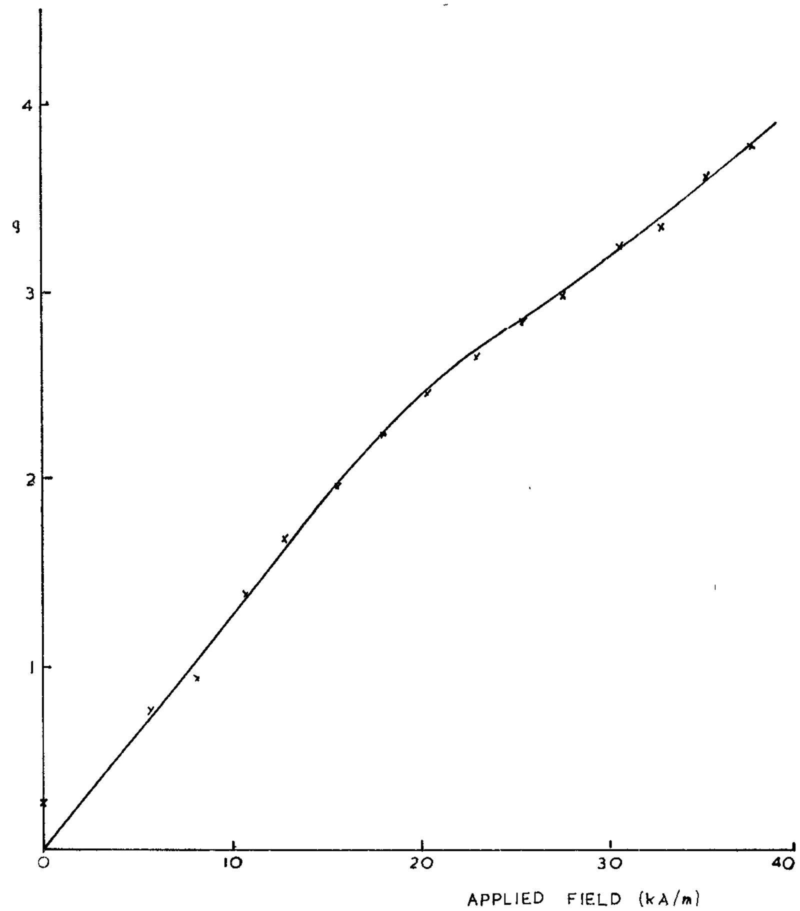

Fig. 5. Gyrator conductance of stripline circulator versus direct magnetic field.

Using the above relations, z becomes

$$z = \frac{\tan \frac{\theta_{+1}}{2}}{\sqrt{3}} \tag{13}$$

and

$$g = \frac{1}{z} = \sqrt{3} \cot \frac{\theta_{+1}}{2}. \tag{14}$$

It is now possible to eliminate θ_{+1} between (8) and (14). This gives a relation between r and g.

The result in terms of the VSWR r is

$$r = \frac{|3 + g^2| + |1 - g^2|}{|3 + g^2| - |1 - g^2|}. \tag{15}$$

For g larger than unity this function is

$$g = \sqrt{2r - 1}. \tag{16}$$

For g smaller than unity it is

$$g = \sqrt{\frac{2 - r}{r}}. \tag{17}$$

The relation between g and r is shown in Fig. 2.

Whether g is larger or smaller than unity is determined by whether there is a voltage maxima or minima at the load. For a demagnetized junction the gyrator conductance is zero.

The experimental data superimposed on Fig. 3 were obtained on a 2-GHz stripline circulator. They were obtained by simultaneously making the measurement described in [1]–[4] which consists of first decoupling one port and directly measuring the gyrator conductance, and then by making the one derived here which consists of measuring the VSWR in the input transmission line of the same port with the transmission lines of the other two ports terminated in their characteristic impedance. The results show good agreement between theory and experiment.

IV. Mode Charts

One definition of an electromagnetic resonance within a junction is the minimization of the reflection coefficient S_{11}. For a reciprocal 3-port junction with $s_1 = s_{-1} = s_{+1}$ the reflection coefficient is

$$S_{11} = \frac{s_0 + 2s_1}{3}. \tag{18}$$

This relation can be effectively applied in practice to study the mode spectrum of junctions. Using this principle it is only necessary to measure the frequency at which the reflection coefficient of the junction passes through a minimum.

With $s_0 = -1$ the minimum value for S_{11} occurs with $s_1 = -s_0$

$$|S_{11}| = \left|\frac{s_1}{3}\right| = \frac{1}{3}. \tag{19}$$

Reference [11] gives the mode chart obtained in the case of an E-plane junction using this technique.

V. Measurement of Split Resonant Frequencies of Junction Circulators

An essential quantity which determines the behavior of junction circulators is the split frequencies of the device. One simple way in which these can be measured is by determining the two frequencies at which the VSWR passes through 2:1. These two frequencies then coincide with the split frequencies of the device. The modes need not be distinguishable to obtain this result. This result can be shown with reference to the eigenvalue diagrams shown in Fig. 3. These illustrations show the eigenvalue arrangement of an arbitrarily magnetized junction at ω_0, ω_{+1}, and ω_{-1}. It is immediately apparent from these diagrams that for a lossless junction the magnitude of the reflection coefficient S_{11} has the value of $S_{11} = \frac{1}{3}$ at both ω_{+1} and ω_{-1}.

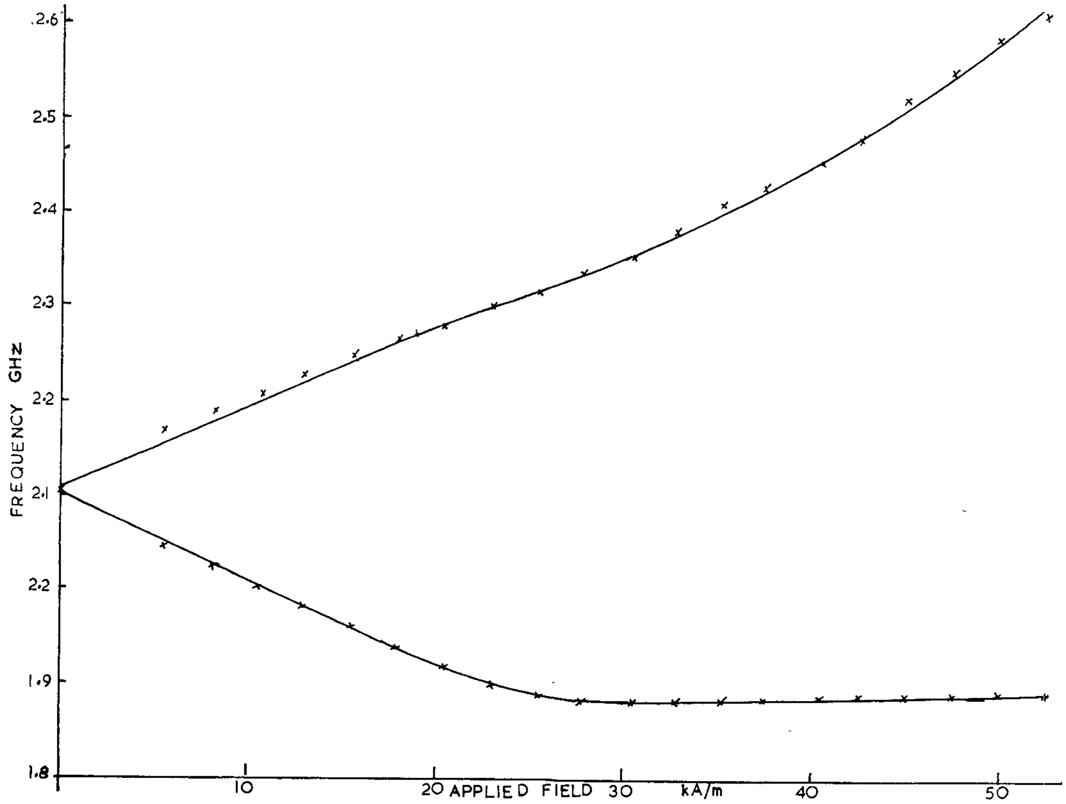

Fig. 6. Split frequencies of stripline junction circulator versus direct magnetic field.

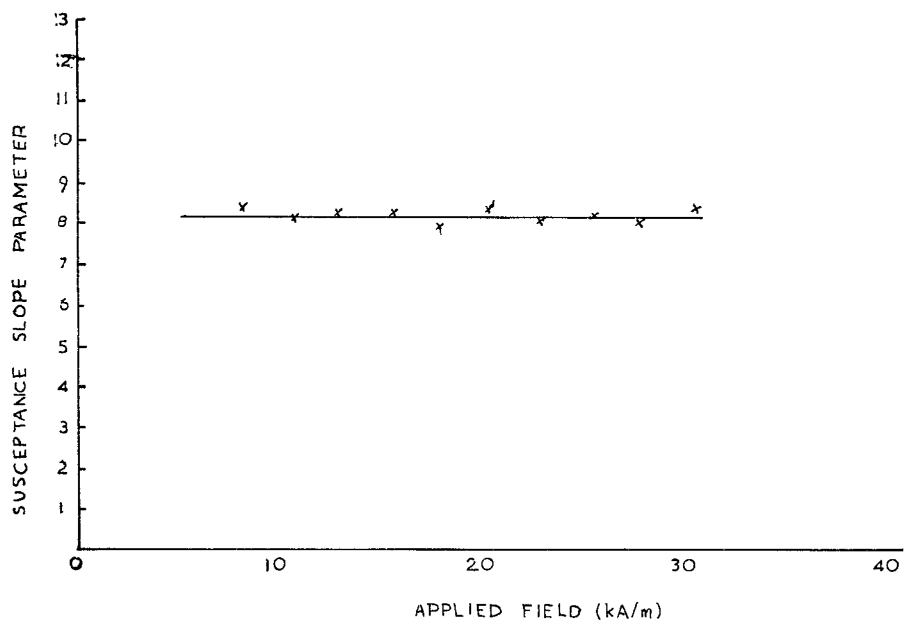

Fig. 7. Susceptance slope parameter of stripline circulator versus direct magnetic field.

At ω_{+1}, (4) gives

$$|S_{11}| = \left|\frac{s_{-1}}{3}\right| = \frac{1}{3}. \tag{20}$$

Similarly, at ω_{-1} one has

$$|S_{11}| = \left|\frac{s_{+1}}{3}\right| = \frac{1}{3}. \tag{21}$$

It was shown in the last section that this condition also coincides with the degenerate resonant frequencies of the the demagnetized junction.

It is further observed that the reflection coefficient at ω_{+1} is s_{-1} and the one at ω_{-1} is s_{+1}. It is therefore possible to measure these quantities have already been described in the text.

If there are losses within the junction, the return loss at the center frequency of the demagnetized junction will depart from its ideal value of 9.5 dB. It can then be shown that, provided the magnetic splitting is small, the two split frequencies now coincide with the actual value of the return loss obtained from the demagnetized junction instead of the ideal one. At large direct magnetic fields it is possible to observe resonance loss for one of the split modes.

The split frequencies are also related to the loaded Q-factor by (1). This allows the loaded Q-factor to be determined.

VI. Susceptance Slope Parameter of Partially Magnetized Junction

One way in which the susceptance slope parameter of a partially magnetized junction may be obtained is by using the universal admittance equation of a junction given by (2).

The susceptance slope parameter is immediately obtained from this last equation by independently measuring the input admittance and the two split frequencies. Methods of measuring these last two quantities have already been described in the text.

It is also observed from this last equation that g is an increasing function of the magnetic field as long as the splitting of the resonant modes is widening.

VII. Experimental Results

This section gives experimental results obtained on a below-resonance stripline circulator using the techniques developed here. The schematic of the stripline circulator investigated is shown in Fig. 4. The junction used here consists of a garnet disk surrounded by a dielectric sleeve. The magnetization of the garnet material used was 0.0500 Wb/m^2 and its dielectric constant was $\epsilon_r = 14.7$. The dielectric constant of the ring was $\epsilon_r = 8.0$. The diameter of the garnet disk was 12.5 mm and the outside diameter of the ring was 25.0 mm. The thickness of the garnet disk was 2.54 mm. The experimental results obtained here are shown in Figs. 5–7. Fig. 5 shows the gyrator conductance of this junction as a function of the direct magnetic field using the method developed in Section III of this text. Fig. 6 shows the two split frequencies of this geometry obtained by using the technique derived in Section V. Finally, Fig. 7 gives the susceptance slope parameter of this junction as a function of the direct magnetic field. This last illustration is obtained by solving the universal gyrator equation for the susceptance slope parameter in terms of the experimental gyrator conductance and split frequencies of the magnetized junction. Fig. 7 also indicates that the susceptance slope parameter of the junction is independent of the direct magnetic field which is as it should be for the dielectric loaded junction used here.

VIII. Conclusions

This short paper has given new simple ways of measuring each of the three parameters which enter into the admittance equation of junction circulators. The methods described require no phase information and are therefore ideally suited for reflectometer-type measurements. All measurements described in this short paper are made in the input transmission line of the junction with the other two ports connected to similar transmission lines terminated in their characteristic impedance. The results obtained here apply to lossless circulators for which the two resonant modes are symmetrically split by the magnetic field, and for which the frequency variation of the third mode can be omitted.

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End Effects of Half-Wave Stripline Resonators

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***Abstract*—The end effects of an open-circuited TEM transmission line make the line electrically longer than its physical length. In this short paper a half-wave resonator of a balanced strip transmission line has been analyzed and the required foreshortening of the line to achieve a prescribed resonance frequency has been calculated. Also the decrease in the characteristic impedance of the stripline caused by the end effects has been determined. The theory is in reasonably good agreement with measurements performed, especially for narrow stripline resonators.**

By using an electrostatic theory Altschuler and Oliner [1] calculated the foreshortening for a strip of infinite width. When the physical length a of the strip exceeds twice the ground-plane spacing b, the foreshortening obtained this way is

$$\Delta a \approx 0.44b. \tag{1}$$

It is natural that the foreshortening is less for a narrow strip. Altschuler and Oliner also gave an empiric formula for the width dependence.

In this short paper a new dynamic method for the calculations of the foreshortening of half-wave stripline resonators will be presented. The method is based on calculations of cavity-backed slot antennas [2], [3]. Fig. 1 shows a cavity for such an antenna. The usual electric wall at the cavity bottom has been changed to a magnetic wall. For a cavity-backed slot antenna an admittance can be defined at the center of the slot. This admittance consists of two parts: one from the exterior region of the cavity and one from the interior region in the cavity. Here we are only interested in the latter part. If we put an identical cavity on the other side of the slot, the slot admittance will be twice the interior admittance of one cavity. On the other hand, the dual configuration of the double cavity-backed slot is just the stripline configuration of Fig. 2. Consequently, by using Babinet's principle we can achieve the input impedance Z_{strip} over the infinitesimal gap in the middle of the stripline resonator, as indicated in Fig. 2, from the interior admittance Y_{slot} of a single cavity

$$Z_{\text{strip}} = \frac{1}{4} 2 Y_{\text{slot}} \frac{\mu_0}{\epsilon_r \epsilon_0} \tag{2}$$

where ϵ_r is the relative dielectric constant of the medium in the cavity, i.e., of the stripline board. Since the medium in the closed box is homogeneous, the field distribution is independent of ϵ_r and so also of the foreshortening. But the impedances and resonance frequencies are of course dependent on ϵ_r. If the magnetic walls in Fig. 2 are moved away from the strip, their influence on the impedance may be neglected.

To get the interior admittance Y_{slot} we use a method [4]–[6] where the different waveguide modes in the cavity build up a proposed electrical field in the slot. At resonance a very good approximation for this field distribution over the slot is

$$E(x, y, 0) = \hat{x} E_0 \cos\left(\pi \frac{y}{a}\right). \tag{3}$$

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